Propagation of Electric Waves Over the Earth

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SYNOPSIS: The comparatively poor transmission of radio waves of two or three hundred meters indicates some sort of selective effect in the atmosphere. Such an effect is found to result from the existence of free electrons in the atmosphere when the magnetic field of the earth is taken into account. In the earth's magnetic field, which is about one-half gauss, this selective effect will occur at a wave length of approximately 200 meters. Ionized hydrogen molecules or atoms result in resonant effects at frequencies of a few hundred cycles, this being outside of the radio range. The paper, however, takes into account the effects of ionized molecules as well as electrons.

The result of this combination is that the electric vector of a wave traveling parallel to the magnetic field is rotated. Waves traveling perpendicular to the magnetic field undergo double refraction. Critical effects are observed in rotation, bending of the wave and absorption at the resonant frequency. The paper develops the mathematical theory of these phenomena and gives formulas for the various effects to be expected.

THE problem of the propagation over the earth of electromagnetic waves such as are used in radio communication has attracted the attention of a number of investigators who have attacked the problem along somewhat different lines, with the purpose of offering an explanation of how electromagnetic waves can affect instruments at a great distance from the source in spite of the curvature of the earth. No attempt will be made here to describe adequately the various theories, but we remark that the theories of diffraction around a conducting sphere in otherwise empty space did not give satisfactory results and led to the necessity for the invention of a hypothetical conducting layer (Heaviside layer) whose aid is invoked to confine the wave between two concentric spherical shells. In many cases this Heaviside layer was considered to have the properties of a good conductor and it was supposed that a beam of short waves, for example, might be more or less regularly reflected back to the earth. The high conductivity of this layer was supposed to be due to the ionizing action of the sun or of particles invading the earth's atmosphere from outside and producing in the rarefied upper atmosphere a high degree of ionization. The differences in transmission during day and night and the variations which occur at sunrise and sunset were supposed to be due to the different ionizing effects of the sun's rays appropriate to the different times of day. The explanation of the phenomenon of "fading" or comparatively rapid fluctuations in the intensity of received signals could then be built up on the assumption of irregularities in the Heaviside layer producing either interference between waves arriving by different paths or reflection to different points on the earth's surface. The principal difficulty in

this explanation is the necessity for rather high conductivity to account for the propagation of waves to great distances without large absorption.

In 1912 there appeared an article by Eccles [1] in which the bending of waves around the surface of the earth was explained on the basis of ions in the upper atmosphere which became more numerous as the vertical height increased and thereby decreased the effective dielectric constant which is a measure of the velocity of propagation of the wave. In this case the velocities at higher levels will be slightly greater than the velocities at lower levels, which will result in a bending downward of the wave normal and a consequent curvature of the wave path to conform to the curvature of the earth. In order to produce this effect without absorption the ions must be relatively free. If they suffer many collisions during the period of a wave, energy will be absorbed from the wave and pass into the thermal agitation of the molecules. Thus absorption of the wave can be computed provided the nature of the mechanism is understood thoroughly.

Sommerfeld and others have worked out the effect of the imperfect conductivity of the ground upon the wave front and such computations lead to a prediction that the electric vector in the wave near the ground will be tilted forward and thus have a horizontal component. This effect of imperfect conductivity is usually given as the cause of the large electromotive force which is induced in the so-called "wave antenna." This effect, however, apparently does not lead to an explanation of the bending of waves around the earth.

There has recently appeared an article by Larmor [2] in which the idea of a density gradient of ions or electrons is developed further to explain the bending of waves around the earth without a large absorption. This paper, as well as that of Eccles, leads to the conclusion that long radio waves will be bent around the earth, and that the effect increases as the square of the wave length, becoming vanishingly small for very short waves.

The large amount of data now available from both qualitative and quantitative observations of radio transmission shows that the phenomena may be more complicated than would be indicated by these theories. It is found that very long waves possess a considerable degree of stability and freedom from fading and that as the wave length decreases the attenuation and the magnitude of fluctuations increases until for a wave length of the order of two or three hundred

[1] Proc. Roy. Soc., June, 1912.

[2] *Phil. Mag.*, Dec., 1924.

meters there is great irregularity in transmission so that reliable communication over land for distances as short as 100 miles is not always possible even with large amounts of power. With decreasing wave length we find also variations in apparent direction of the wave. On the other hand, as the wave length is decreased still further we find, sometimes, rather surprising increases in range and stability. The nature of the fading changes, becoming more rapid, and the absorption in many cases seems to decrease. This peculiarity of wave transmission must be explained in a satisfactory theory. In addition to the apparent selective effect just mentioned, some observations indicate that there are often differences between east and west and north and south transmission at all wave lengths.

The various irregularities in radio transmission, and particularly the apparently erratic and anomalous behavior of electromagnetic waves occurring in the neighborhood of a few hundred meters wave length seem to indicate that as the wave length is decreased from a value of several kilometers to a value of a few meters some kind of selective effect occurs which changes the trend of the physical phenomena. These considerations have suggested to us the possibility of finding some selective mechanism in the earth's surface or in the atmosphere which becomes operative in the neighborhood of 200 meters. A rather superficial examination of the possibility that such a selective mechanism may be found in a possible distribution of charged particles in the atmosphere has resulted in the conclusion that a selective effect of the required kind cannot be produced by such a physical mechanism. There is, however, in the earth's atmosphere—in addition to distributions of ions—a magnetic field due to the earth, which in the presence of ions will have a disturbing effect upon an electromagnetic wave. As is well known, a free ion moving in a magnetic field has exerted upon it, due to the magnetic field, a force at right angles to its velocity and to the magnetic field. If the ion has impressed upon it a simple periodic electric force, it will execute a free oscillation together with a forced oscillation whose projection on a plane is an ellipse which is traversed in one period of the applied force. The component velocities are linear functions of the components of the electric field and at a certain frequency, depending only upon the magnetic field and the ratio $\frac{e}{m}$ of the ion, become very large unless limited by dissipation. This critical frequency is equal to $\frac{He}{2\pi mc}$ if H is measured in electromagnetic units and e in electrostatic units. It is the same as the frequency of free

oscillation. For an electron in the earth's magnetic field (assumed to have a value of 1/2 gauss) this resonant frequency is 1.4×10^6 cycles, corresponding to a wave length of 214 meters.[3] We thus have an indication that some at least of the phenomena of transmission at the lower wave lengths may be explained by taking into account the action of the earth's magnetic field upon electrons present in the earth's atmosphere and acted upon by the electric field of the wave. This frequency occurs at approximately the position in the spectrum at which the peculiar effects already mentioned occur. The next resonant frequency which would be encountered would be due to the hydrogen ion which has a ratio, $\frac{e}{m}$, equal to $\frac{1}{1800}$ that of the electron. The resonant frequency of this ion is only 800 cycles and certainly can have no sharply selective effect in the propagation of electromagnetic waves over the earth. We have, therefore, worked out the consequences of the assumption that we have in the upper atmosphere two controlling factors influencing the propagation of electromagnetic waves in the radio range, namely, free electrons and ions together with the earth's magnetic field. The electrons will be dominant in their effects in the neighborhood of the resonant frequency and perhaps above, while the heavy ions will affect the wave at all frequencies and, if much more numerous, may be controlling at frequencies other than the critical one. In working out this theory it is assumed that there are present in the earth's atmosphere free electrons and ions. At high altitudes these are capable, on the average, of vibrating under the influence of the electromagnetic field through several complete oscillations before encountering other ions or neutral atoms. At low altitudes this assumption will not hold, the collisions being so numerous that the importance of the resistance term in the equations of motion becomes much greater. In either case the ions have no restoring forces of dielectric type. The motion of the electron or ion constitutes a convection current which reacts upon the electromagnetic wave and changes the velocity

[3] This frequency does not depend upon the direction of the field, and is practically constant over the earth's surface.

On March 7, after this paper had been written, the February 15 issue of the Proceedings of the Physical Society of London arrived in New York. In this journal there was a discussion on ionization in the atmosphere in which Prof. E. V. Appleton suggested, in an appendix, that the earth's magnetic field acting upon electrons would change the velocity of a wave and produce rotation. A calculation of the critical frequency was given in which, however, only the horizontal component of the earth's field was used, resulting in an incorrect value for the critical frequency, namely less than half the actual value. If the complete equations are written down it is evident at once that the total field is involved in the critical frequency, no matter what may be the direction of propagation.

of propagation of the wave. This is, in fact, the basis for the explanation of the optical properties of transparent and absorbing media and also of media which show magnetic or other rotatory powers. Due to collisions and recombinations, energy will pass continuously from the electromagnetic field and increase the energy of agitation of neutral molecules. Since this process is irreversible it accounts for absorption of energy from the wave.

Assume an electron or ion of charge e and mass m moving with velocity $\boldsymbol{v}$ and acted upon by an electric field $\boldsymbol{E}$ and the earth's magnetic field $\boldsymbol{H}$. The equation of motion of the free ion will be

$$\frac{m}{e}\dot{\boldsymbol{v}}=\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{H}$$

or

$$a\dot{\boldsymbol{v}}=\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{h} \tag{1}$$

in which $\boldsymbol{h}$ is written for $\dfrac{\boldsymbol{H}}{c}$ and a for m/e. (When we come to consider absorption it will be necessary to generalize a into $a\left(1-i\dfrac{r}{mn}\right)$ to include a resisting force, rv, proportional to the velocity.)

The total current is given by

$$4\pi\boldsymbol{I}=\dot{\boldsymbol{E}}+\sum 4\pi Ne\,\boldsymbol{v}. \tag{2}$$

In these equations and the following we are using Gaussian units and the summation refers to different kinds of ions.

In order to avoid a complicated mathematical treatment, which, however, is not difficult to carry through if necessary, it will be assumed that the magnetic field $\boldsymbol{H}$ is along the axis of z. When more general results are required, they will be stated. All time variables are assumed periodic with a frequency $\dfrac{n}{2\pi}$, so that $\dfrac{\partial}{\partial t}=in$.

Solving equation (1) for the components of $\boldsymbol{v}$ we find, for each type of ion:

$$v_1=\frac{ina\,X+h\,Y}{h^2-a^2n^2},$$

$$v_2=\frac{-hX+ina\,Y}{h^2-a^2n^2},$$

$$v_3=\frac{Z}{ina},$$

from which it appears that a resonance frequency occurs for

$$n = \frac{h}{a} = n_o.$$

Since e/m for the electron is $-1.77c \times 10^7$, the earth's magnetic field of about 1/2 gauss will produce a resonance frequency at 1.4×10^6 corresponding to a wave length of 214 meters, while all heavier ions have resonance frequencies far outside the spectral region to be considered.

The assumption that the components of the ionic motion are simple harmonic, in spite of the fact that the motion of the ion is rather complicated, is justified as follows. From (1) we find that the velocity of an ion (r), say $\boldsymbol{v}_r$ is made up of the complementary solution, $\boldsymbol{v}_r'$ and the particular solution $\boldsymbol{v}_r'' = f(\boldsymbol{E})$. The latter depends upon the impressed force $\boldsymbol{E}$, while the former has constants of integration determined by the position and motion of the ion at the last collision. The complete current is thus

$$\boldsymbol{I} = \frac{1}{4\pi}\dot{\boldsymbol{E}} + \sum e\boldsymbol{v}_r' + Nef(\boldsymbol{E}).$$

The second term, however, averages out over a large number of ions since the initial conditions are random;[4] hence, as far as the effect upon wave propagation is concerned, we may treat all quantities as periodic.

Following the usual procedure for the investigation of the propagation of waves in media of this kind, we shall rewrite equation (2) in terms of the components of the electric field, thus for each type of ion:

$$\begin{aligned}
4\pi I_1 &= \left(1 + \frac{\sigma N}{n_o^2 - n^2}\right)\dot{X} - i\,\frac{\sigma N \frac{n_o}{n}}{n_o^2 - n^2}\,\dot{Y} = \epsilon_1 \dot{X} - i\alpha \dot{Y}, \\
4\pi I_2 &= i\,\frac{\sigma N \frac{n_o}{n}}{n_o^2 - n^2}\,\dot{X} + \left(1 + \frac{\sigma N}{n_o^2 - n^2}\right)\dot{Y} = i\alpha \dot{X} + \epsilon_1 \dot{Y}, \qquad (3) \\
4\pi I_3 &= \left(1 - \frac{\sigma N}{n^2}\right)\dot{Z} = \epsilon_2 \dot{Z},
\end{aligned}$$

in which $\frac{4\pi e}{a} = \sigma$, or 3.2×10^9 for an electron and $3.2 \,.\, 10^9 \, \frac{m}{M}$ for an ion of mass M. In order to avoid complicated formulas, the summations which must be carried in equations (3) to take account

[4] It is here assumed that the mean time between collisions is large compared to $\frac{1}{n}$.

of the effect of ions of different kinds have been omitted, but it is to be understood that the dielectric constants ϵ, α, etc., are built up from the contributions of all types of ions. Thus for an ion of mass M we must put $\sigma \frac{m}{M}$ for σ, $n_o \frac{m}{M}$ for n_o, in equations (3).

The effective dielectric constant, instead of being unity, has thus the structure:

$$(\epsilon)=\begin{pmatrix} \epsilon_1 & -i\alpha & 0 \\ i\alpha & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{pmatrix}$$

and we may write equation (2) as

$$4\pi \boldsymbol{I}=(\epsilon)\dot{\boldsymbol{E}}$$

which has the significance of the scalar equations (3). Thus $\boldsymbol{I}$ is a linear vector function of $\boldsymbol{E}$ and the operator (ϵ) is skew symmetric, indicating a rotatory effect about the axis of z.

(The general case in which h has the three components $(h_1\ h_2\ h_3)$ results in a dielectric constant having the structure

$$(\epsilon)=\begin{pmatrix} \epsilon_1 & -\beta_3-i\alpha_3 & -\beta_2+i\alpha_2 \\ -\beta_3+i\alpha_3 & \epsilon_2 & -\beta_1-i\alpha_1 \\ -\beta_2-i\alpha_2 & -\beta_1+i\alpha_1 & \epsilon_3 \end{pmatrix}$$

of which the above is a special case. With this value of (ϵ) the equation (4) below contains the *general* solution of our problem.)

Let $\boldsymbol{H}_1$ be the magnetic force associated with $\boldsymbol{E}$ in the wave so that

$$c \operatorname{curl} \boldsymbol{H}_1=(\epsilon)\ \dot{\boldsymbol{E}}$$

$$c \operatorname{curl} \boldsymbol{E}=-\dot{\boldsymbol{H}}_1.$$

Eliminating $\boldsymbol{H}_1$ from these equations we get

$$-\nabla^2\boldsymbol{E}+\nabla \operatorname{div} \boldsymbol{E}=\frac{n^2}{c^2}(\epsilon)\boldsymbol{E} \tag{4}$$

or in scalar form

$$-\nabla^2 X+\frac{\partial}{\partial x} \operatorname{div} \boldsymbol{E}=\frac{n^2}{c^2}(\epsilon_1 X-i\alpha Y),$$

$$-\nabla^2 Y+\frac{\partial}{\partial y}\operatorname{div} \boldsymbol{E}=\frac{n^2}{c^2}(i\alpha X+\epsilon_1 Y), \tag{5}$$

$$-\nabla^2 Z+\frac{\partial}{\partial z}\operatorname{div} \boldsymbol{E}=\frac{n^2}{c^2}(\epsilon_2 Z).$$

These equations for the propagation of light in magnetically active substances have been given by Voigt, Lorentz, Drude and others and form the basis of the explanation of optical phenomena in such substances. As applied to optics, they are worked out, for example, in Drude's "Optics" (English translation), page 433. As applied to this problem, they assume either that the motion of the ions is unimpeded or that the resistance to the motion may be expressed as a constant times the velocity, which, as explained later. may be done in this case. We shall work out some comparatively simple cases and point out the conclusions to be drawn from them.

Consider first a plane polarized ray having its electric vector parallel to the magnetic field and moving in the xy plane; for example parallel to x. In this case the electric vector is a function of x and t only of the form

$$Z=Z_o\,\epsilon^{in\left(t-\frac{\mu x}{c}\right)}$$

in which $\frac{c}{\mu}$ is the velocity of the wave. Substituting in the general equations (5) we find that

$$\mu^2=1-\Sigma\frac{\sigma_i N_i}{n^2}. \tag{6}$$

The velocity of propagation is thus a function of the frequency and of the density N. This particular case corresponds to that treated by Eccles and Larmor in the papers cited. It will be noted that the velocity is greater for long waves than for short waves and that if N is a function of distance from the surface of the earth, the velocity will vary in a vertical direction, causing a curvature of the rays as worked out by the authors mentioned. In this particular case, however, which corresponds completely in practice to conditions obtaining over only a limited area of the earth's surface, the greatest effect is produced on the longer waves. Since electromagnetic waves are in general radiated from vertical antennas so that the electric vector is vertical, this case would correspond to the condition of transmitting across the north or south magnetic poles of the earth.

The second case to be considered is that of propagation along the direction of the magnetic field. In this case X and Y are functions

of z and t and the appropriate solutions of the fundamental equations (5) are

$$X' = A \cos n\left(t - \frac{\mu_1 z}{c}\right),$$

$$Y' = -A \sin n\left(t - \frac{\mu_1 z}{c}\right), \quad \mu_1^2 = \epsilon_1 + \alpha,$$

$$X'' = A \cos n\left(t - \frac{\mu_2 z}{c}\right),$$

$$Y'' = A \sin n\left(t - \frac{\mu_2 z}{c}\right), \quad \mu_2^2 = \epsilon_1 - \alpha.$$

which represent two oppositely circularly polarized components traveling with the different velocities $\frac{c}{\mu_1}$ and $\frac{c}{\mu_2}$. The plane of polarization is rotated through an angle of 2π in a distance given by

$$\frac{z_o}{\lambda} = \frac{\epsilon_1}{\alpha}.$$

The third case to be considered is that of propagation at right angles to the magnetic field, say in the direction of x. For this case equations (5) become:

$$X = \frac{i\alpha}{\epsilon_1} Y$$

$$-\frac{c^2}{n^2}\nabla^2 Y = \left(\epsilon_1 - \frac{\alpha^2}{\epsilon_1}\right) Y$$

$$-\frac{c^2}{n^2}\nabla^2 Z = \epsilon_2 Z,$$

of which the solutions are

$$X = \frac{i\alpha}{\epsilon_1} Y_o \epsilon^{in\left(t - \frac{\mu_1 x}{c}\right)}$$

$$Y = Y_o \epsilon^{in\left(t - \frac{\mu_1 x}{c}\right)} \qquad \mu_1^2 = \epsilon_1 - \frac{\alpha^2}{\epsilon_1}.$$

$$Z = Z_o \epsilon^{in\left(t - \frac{\mu_2 x}{c}\right)}.$$

The first of these is merely the (usually small) component of field required to make the total current solenoidal, that is, to balance the

convection of electrons. The last two show that the plane polarized ray whose electric vector is parallel to H will travel with the velocity $\frac{c}{\mu_2}$ while the one whose electric vector is at right angles to this direction and to the direction of propagation will travel at a different speed, $\frac{c}{\mu_1}$. There is thus double refraction.

Bending of the rays. If μ is the index of refraction, which is a function of the space variables, the curvature of the ray having this index is $\frac{1}{\mu}\frac{d\mu}{ds}$ where s is taken perpendicular to the direction of the ray. Since μ is practically unity except at the critical frequency, this curvature is $1/2\ d\,\mu^2/ds$. In order that the ray should follow the curvature of the earth it is clear that μ must decrease at higher altitudes; that is, $\frac{d\mu^2}{ds}$ must be negative.

We shall work out the curvatures for the special cases considered. (The first case has been given above and was worked out in the papers cited). For the case of propagation along H, the two circularly polarized beams have indices given by

$$\mu_1^2=\epsilon_1+\alpha=1+\frac{\sigma N}{n^2}\ \frac{1}{\omega-1}, \tag{7}$$

$$\mu_2^2=\epsilon_1-\alpha=1-\frac{\sigma N}{n^2}\ \frac{1}{\omega+1}, \tag{8}$$

$$\left(\omega=\frac{n_o}{n}\right).$$

We are interested in the values of $1/2\,\frac{d\mu^2}{ds}$ in which N and h are functions of distance s and also of the time. These come out to be

$$C_1=\frac{\sigma}{2n_o^2}\left[\frac{\omega^2}{\omega-1}\ \frac{dN}{ds}-\frac{\omega^3}{(\omega-1)^2}\ \frac{N}{h}\ \frac{dh}{ds}\right], \tag{9}$$

$$C_2=\frac{\sigma}{2n_o^2}\left[\frac{-\omega^2}{\omega+1}\ \frac{dN}{ds}+\frac{\omega^3}{(\omega+1)^2}\ \frac{N}{h}\ \frac{dh}{ds}\right]. \tag{10}$$

A striking fact shown by these formulae is that the curvatures of the two rays are in general different. A limited beam entering an ionized medium along a magnetic meridian will be split into two which will traverse different paths. Thus we should expect to find,

occasionally, a circularly polarized beam at the receiver due to the fact that the receiving instrument is located at a point toward which one of the beams is diverted after having passed through an upper ionized layer. This is now being investigated experimentally. It is clear that, although the two components do not in general travel over the same path, both may eventually arrive at the same receiver. The first ray, however, may have penetrated much higher in the atmosphere than the other, that is, to a level at which $\frac{dN}{ds}$ has the proper negative value to cause it to return to earth.

For long waves, these curvatures become:

$$C_1=\frac{\sigma\omega}{2n_o^2}\left[+\frac{dN}{ds}-\frac{N}{h}\frac{dh}{ds}\right], \tag{11}$$

$$C_2=\frac{\sigma\omega}{2n_o^2}\left[-\frac{dN}{ds}+\frac{N}{h}\frac{dh}{ds}\right]. \tag{12}$$

Hence a limited beam of long waves entering this medium would tend to split into two of opposite polarization and traverse different paths. In the special case for which $\frac{1}{N}\frac{dN}{ds}=\frac{1}{h}\frac{dh}{ds}$ throughout the medium, there will be no such separation of the beam.

For very short waves

$$C_1=\frac{\sigma}{2n_o^2}\left[-\omega^2\frac{dN}{ds}-\omega^3\frac{N}{h}\frac{dh}{ds}\right], \tag{13}$$

$$C_2=\frac{\sigma}{2n_o^2}\left[-\omega^2\frac{dN}{ds}+\omega^3\frac{N}{h}\frac{dh}{ds}\right]. \tag{14}$$

Hence if the most effective cause of refraction is the variation in the ionic density both components tend to remain together and to travel with a rotation of the plane of polarization. If variation in the magnetic field is appreciable the two components tend to diverge as in the case of long waves.

For propagation at right angles to H, say along x, we have

$$\mu_1^2=\epsilon_2=1-\frac{\sigma N}{n^2}, \tag{15}$$

$$\mu_2^2=\epsilon_1-\frac{\alpha^2}{\epsilon_1}. \tag{16}$$

The bending of the plane polarized component having the index μ_1 shows no selective effects, being simply

$$C_1 = -\frac{\sigma}{2n^2}\frac{dN}{ds} \tag{17}$$

and is appreciable only for long waves unless N is very large. For the other component we find:

$$C_2 = \frac{\sigma}{2n_o^2}\cdot\frac{\omega^2}{\omega^2-1}\,\frac{1-\dfrac{2\sigma N}{n_o^2}\omega^2-\dfrac{\sigma^2N^2}{n_o^4}\dfrac{\omega^2}{\omega^2-1}}{\left(1+\dfrac{\sigma N}{n_o^2}\dfrac{\omega^2}{\omega^2-1}\right)^2}\frac{dN}{ds} \tag{18}$$

where, in order to simplify the formula, only the term containing $\frac{dN}{ds}$ has been included. This applies to ions of one kind.

For long waves these two curvatures become

$$C_1 = -\frac{\sigma}{2n_o^2}\omega^2\frac{dN}{ds}, \tag{19}$$

$$C_2 = \frac{\sigma}{2n_o^2}\left(1-\frac{2\sigma N}{n_o^2}\omega^2\right)\frac{dN}{ds}. \tag{20}$$

These formulas show that the first curvature is always in the same direction for a given value of $\frac{dN}{ds}$, while the second curvature, which is that of the electric vector perpendicular to the magnetic field, is, for very long waves, in the same direction as C_1 but, as the wave length is decreased or N increased, reverses in sign and becomes opposite to C_1. As an example, if $N=10$, for 6 kilometer waves the curvatures are opposite, so that if the first component tends to bend downward the second will tend to bend upward; while if $N=100$, for the same wave length both curvatures have the same sign and the second is five times as large as the first.

For extremely short waves the two curvatures are equal as they obviously should be, since the magnetic field can then have no effect.

In transmitting from New York to London, for example, waves travel approximately at right angles to the magnetic field, which in this latitude has a dip of about 70°. If we assume a plane polarized ray starting out with its electric vector vertical, the component parallel to the magnetic field will be the larger and will be subject to the curvature C_1 above, while the smaller component will be affected

by the magnetic field and will have the curvature C_2. The two components into which the original wave is resolved will travel with different velocities. It is clear that when the distribution of ions in the upper atmosphere is changed by varying sunlight conditions, the resulting effect at a receiver is likely to vary considerably. Some of the possibilities will be discussed later.

Rotation of the plane of polarization. It has been shown that in the second case, namely transmission along the magnetic field, there will be a rotation of the plane of polarization of the wave. This rotation is such that the wave is rotated through a complete turn in a distance given by

$$z_o = \frac{2\pi c}{n_o} \frac{1+\dfrac{\sigma N}{n_o^2}\dfrac{\omega^2}{\omega^2-1}}{\dfrac{\sigma N}{n_o^2}\dfrac{\omega^2}{\omega^2-1}}. \tag{21}$$

It is interesting to note that the distance in which a long wave rotates through 2π approaches the constant value $\dfrac{2\pi c n_o}{\sigma N}$ as the wave length increases and that for very short waves the rotation of the plane of polarization tends to vanish with the wave length.

Absorption. When an electron strikes a massive neutral atom the average persistence of velocities is negligible and in the steady state of motion of electrons and neutral molecules the element of convection current represented by an impinging electron will be neutralized, so far as the wave is concerned, at every collision. Of the energy which has been put into this element of convection current since the last collision, a part will be spent in accelerating neutral molecules, part will go to increase the average random velocity of the electron and a part will appear as *disordered* electromagnetic radiation. Thus, as far as the wave is concerned, the process of collision with massive neutral molecules is irreversible even if the molecules are elastic, and all the energy picked up by the electron from the wave between collisions is taken from the wave at the next collision. Exactly the same state of affairs would exist if at each collision the electron recombined with a molecule and a new electron were created with zero or random velocity. Thus for massive molecules for which we can neglect the persistence of electron velocities the effect upon the wave is exactly the same whether the collision is elastic or inelastic.

These conclusions are verified by the results of two different computations which we have made of the resistance term, rv, in equation

of motion of the electron. Consider in the first place a mixture of electrons and massive neutral molecules, assumed perfectly elastic, in which the persistence of velocities of the electrons after collision is negligible. If an electric field $X\epsilon^{int}$ operates in the x direction and if the state of motion is a steady one, we can compute the energy w taken from the wave by a single electron at any time after a collision at the time t_1 and before the next collision. Let this time after t_1 be τ. If the mean frequency of collisions is f, the time τ between collisions will be distributed according to the law

$$f\epsilon^{-f\tau}$$

and we shall obtain the mean energy taken from the wave per collision by multiplying w by the above expression, integrating from zero to infinity with respect to τ and then performing an average over all the times t_1. The result of this is that the mean energy loss per collision is simply

$$w=\frac{e^2X^2}{2mn^2}\,\frac{n^2}{f^2+n^2}$$

and consequently the loss per second is f times this. If we equate this to rv^2, which is also the rate at which energy is being dissipated, we find that $r=mf$, which is therefore the resistance term to be inserted in the equation of motion of the electron.

If the convection current is carried partly by heavier ions, it will not be annulled at each collision and all the energy derived from the field will not be lost on impact.

The foregoing computation assumes as obvious that energy is lost from the wave at a rate equal to the number of collisions times the average energy which the electron takes from the wave between collisions. The second method is somewhat more general. The mean velocity at a time t is found for electrons which collided last in an interval at t_1. This is evidently a function of the velocity persisting through the last collision and hence of the average velocity before the impact; so that if the average velocity before collision was v, that after impact would be $\delta\, v$, in which δ is a number less than unity, depending on the relative masses and the nature of the collision. Averaging for all values of t_1 before t and using the same law of distribution assumed above, the mean velocity of the ions since the last collision is obtained. By comparison with the solution obtained for the velocity of forced oscillation in which the resistive force is rv, we find that $r=mf(1-\delta)$. For the special case of electrons, δ may be taken equal to zero, hence $r=mf$. For the case

of very heavy ions colliding with light neutral molecules, $r=o$, since $\delta=1$. For equal masses δ would be about one half, hence $r=\frac{1}{2}\,mf$.

Since the resistance factor r is equal to mf, in order to include the effect of attenuation of the wave, we must replace a by

$$a\left(1-i\frac{f}{n}\right).$$

If, as usual, we assume a wave proportional to

$$\epsilon^{\frac{-nk\mu x}{c}} \quad \epsilon^{in\left(t-\frac{\mu x}{c}\right)}$$

the equations (5) show that, in order to calculate the value of the absorption constant k, we must put

$$\mu^2(1-ik)^2=\epsilon$$

in which ϵ is the generalized dielectric constant appropriate to the case considered. We have worked out in this way the absorption for the various cases treated above with the following results.

In the case in which there is either no magnetic field or the magnetic field is parallel to the direction of the electric vector, we find

$$k=\frac{\sigma N}{2n_o^2}\omega^2 \; \frac{f/n}{1+f^2/n^2}.$$

This formula for absorption applies (for electrons) for any value of f or n. Thus near the surface of the earth where the collision frequency f is of the order of 10^9, the fraction $\frac{f}{n}$ will be large even for rather short waves. As we go higher in the atmosphere this ratio decreases for a given wave frequency until at a height for which $\frac{f}{n}=1$ we encounter the maximum absorption *per electron.* Above this level $\frac{f}{n}$ and consequently the absorption per electron decreases. For ions other than electrons the resistance will be somewhat different from mf, depending upon the ratio of the masses, and a corresponding change must be made in the above statement.

In this paper we are considering only the effects which take place at heights above that for maximum absorption so that, generally speaking, $\frac{f}{n}$ will be small or at least less than unity. This approximation will be used in computing the absorption constants which follow.

As an example of the nature of this approximation, at a height of about 100 kilometers, we may expect an atmospheric pressure of 10^{-5} standard and a corresponding collision frequency of the order of 10^5. Thus for very long waves of frequency 40,000 cycles per second we still have $\frac{f}{n}=.4$, while at the critical frequency $\frac{f}{n}$ is only 1/100.

The computation of the collision frequency for electrons is rather involved because of the peculiar nature which such a collision may have and because it probably is not permissible to assume thermal equilibrium with the molecules of the gas. The processes of ionization and recombination will also lead to complications. Probably the most significant information would be the number of electron free paths per second for unit volume.

The question of the behavior of waves in or below the layer of maximum absorption per ion is a somewhat different one and belongs properly in another paper.

For the case of transmission along a magnetic meridian the oppositely circularly polarized rays have the absorption constants:

$$k_1=\frac{\sigma N}{2n_o^2}\frac{\omega^2 f/n}{(\omega-1)^2+(f/n)^2},\qquad k_2=\frac{\sigma N}{2n_o^2}\frac{\omega^2}{(\omega+1)^2}\frac{f}{n}.$$

It will be noted that, at the critical frequency, the first of these waves has the high absorption $\frac{\sigma N}{2n_o^2}\cdot\frac{n}{f}$ and is therefore extinguished in a short distance, while the other wave has a normal absorption constant $\frac{\sigma N}{8n_o^2}\cdot\frac{f}{n}$. Thus for the case of transmission along a meridian at the critical frequency we might expect a receiving station, sufficiently far above the ground, to receive a circularly polarized beam. This would mean that if a loop were used for reception, the intensity of the received signal would be independent of the angle of setting of the loop, provided one diameter of the loop was set parallel to the direction of propagation of the wave. In general, of course, this ideal condition could not be realized due to the disturbing action of the ground and of other conducting or refracting bodies and the most we should expect to receive in practice would be an elliptically polarized beam.

In the third case, namely, that of propagation perpendicular to the direction of the magnetic field, we find that the wave polarized with its electric vector parallel to the magnetic field has the same

absorption as before, namely $\frac{2n_o^2}{\sigma N}\,\omega^2\,\frac{f}{n}$ and the other ray whose complex index of refraction is $\epsilon_1 - \frac{\alpha^2}{\epsilon_1}$ has the absorption constant $\frac{1}{2}\,(k_1+k_2)$ in which k_1 and k_2 are the absorption constants given above for propagation along a magnetic meridian.

At the critical frequency we find, therefore, that the absorption constant is abnormally high and equal to $\frac{\sigma N}{4n_o^2}\cdot\frac{n}{f}$ which is one-half that obtained for the first ray of case 2.

One very striking fact is brought to light by these equations. Thus, referring to the two values of absorption constants for transmission along the magnetic field, we find that for very long waves (for which ω is large) the iònic absorption is very much less with a magnetic field present than without it. This means that in this case and in the next the presence of a magnetic field *assists* in the propagation of an electromagnetic wave by decreasing the absorption. This reduction in absorption may amount to a rather large amount, as may be seen from an inspection of the formula for k_1. For example, if in this case ω is 20, corresponding to 4,000 meter waves, we find that under corresponding conditions the absorption *due to electrons only* is reduced by the magnetic field to 1/400th the value it would have for no magnetic field. Of course, these cases are not directly comparable because the path chosen by the wave would be different in the two cases. It is plausible, however, that the propagation of long waves along the magnetic field may go on with much less attenuation than propagation from East to West over a region in which the magnetic field is nearly vertical, in which case the effect of the magnetic field is largely absent. This conclusion, however, cannot be made in general since a number of other causes are influential in determining the propagation, for example, the bending of the rays, so that it is not certain that transmission over a region in which the magnetic field is vertical is always more difficult than in the other cases.

The reason for the decreased absorption of long waves when the magnetic field can operate (that is, in all cases in which the electric vector is not parallel to the field) is that the velocities acquired by the free electrons are much less for small values of n when the magnetic field is present.

Fading. By this is meant a variation with time of the strength of a received signal at a given point. It is clear that a wave starting

originally with constant amplitude and frequency can be received as one of variable amplitude only if certain characteristics of the medium are variable with the time. So far as the atmosphere is concerned, these characteristics may be the distribution of electrons and heavier ions and the intensity and direction of the earth's magnetic field. If these are functions of the time, the velocities, bending, absorption and rotation of the plane of polarization will all be variable, the amplitude of variation depending upon the variations of N, $\frac{dN}{ds}$, H, $\frac{dH}{ds}$, as well as the frequency of the wave, the effects being in many cases magnified greatly in the neighborhood of the critical frequency. These effects are obviously sufficiently numerous to account for fading of almost any character and suggest a number of experiments to determine the most effective causes. The question of rotation of the plane of polarization, fading and distortion is now being examined experimentally.

From the formulas it is clear that the velocity, curvature and absorption of an electromagnetic wave as well as the rotation of its plane of polarization can all be affected by a time variation in the intensity and direction of the earth's field. An examination of the probable time and space variations of each, however, lead us to the conclusion that these are not of primary importance in determining large amplitude fading except, perhaps, during magnetic storms. One result of the last two years of consistent testing between New York and London at about 60,000 cycles has shown that severe magnetic storms are always accompanied by corresponding variations in the strength of received signals. Thus, although the earth's magnetic field can well exercise a large influence upon the course and attenuation of radio waves, it does not seem likely that its time variation is ordinarily a large contributing cause to fading.

This leaves as the probable principal cause of time variations the number and distribution of ions in the earth's atmosphere. It is impossible in this paper, which is devoted primarily to a development of a theory of transmission involving the earth's magnetic field, to consider adequately all the possibilities resulting from changes in ionic distributions, but some general remarks may be made. Imagine a wave traveling from the source to the receiver. At a short distance from the source the wave front will be more or less regular but as it progresses, due to the irregularities in ionic distribution, the wave front will develop crinkles which become exaggerated as the wave goes on. These crinkles in the wave front will be due to irregularities in the medium and can be obtained by a Huyghen's construction at

any point. If we consider the wave a short distance before it reaches the receiver, we will find regions in which the wave front is concave to the receiver and regions of opposite curvature. Thus at certain portions of the wave front energy will be concentrated toward a point farther on and at other parts will be scattered. The location of these convex or concave portions of the wave in the neighborhood of a given receiving point will be very sensitive to changes in ionic distribution along all the paths of the elementary rays contributing to the effect at the receiver. Hence, if we knew the location and movement of all the ions between the transmitter and the receiver, it would be possible, theoretically, to predict the resultant effect at the latter point.

To explain fading it is essential that there be a time variation in this distribution. It is clear that effects of this kind should be more marked at short waves than at long waves since a region of the medium comparable in dimensions to a wave length must suffer some change in order to produce an effect upon the received signal. If, for example, there were space irregularities in the medium comparable to the wave length, a kind of diffraction effect would be produced at the receiver which would be very sensitive to slight changes in grating space.

A possible cause of irregularity may be found in the passage across the atmosphere of long waves of condensation and rarefaction, each of which results in a change in the density and gradient of the ions, even though the average density remains constant throughout a large volume. If, as seems plausible, the upper atmosphere is traversed by many such atmospheric waves of great wave length, the resulting effect at a given receiving point would be fluctuations in signal strength due to a more or less rapid change in the configuration of the wave front near the receiver.

For radio waves whose length is of the order of a few hundred meters, fading experimentally observed occurs at a rate of the order of one per minute (of course, it is not implied by this statement that there is any regular periodicity to the fading). The pressure wave referred to would travel in the upper atmosphere with a velocity of the order of 300 meters per second at lower levels or 1,000 meters in the hydrogen atmosphere, so that the wave length of these "sound" waves would be of the order of 50 of the radio wave lengths. The irregularities of the medium would thus be of sufficient dimensions with respect to the electromagnetic waves so that one of the characteristics referred to above might be developed. In this way we might explain variations in intensity of the wave at the receiver recurring at intervals of a minute or so.

These effects, of course, might be produced even without a magnetic field but the results of this paper indicate that conditions in the wave front will be complicated still further by a rotation of the electric vector and by the existence of bending and double refraction due to the magnetic field, these effects being exaggerated in the neighborhood of the critical frequency. Due to the magnetic field we have also the possibility of summation effects between components of the wave which were split off by the action of the field and consequently had traveled by different paths at different speeds. It is obviously impossible to make any general statement concerning the nature of the effects which will be produced by this complicated array of causes but future experimental work will, we hope, allow us to estimate the relative importance of the various elements.